MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

TMA1101 – CALCULUS

(All sections / Groups)

12 OCTOBER 2019
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of five pages with **FIVE** questions.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided.
4. **You are required to write proper steps. No calculators are allowed.**

Question 1 (10 marks)

(a) Find the following limits.

[***You must show at least one intermediate step where*** $\lim\limits_{x\to c}$ ***is still needed.***]

(i) $\lim\limits_{x\to 2}\dfrac{x^2-4}{2x^2-3x-2}$

(ii) $\lim\limits_{x\to 5}\dfrac{x-5}{\sqrt{x+20}-5}$

[2.5 marks]

(b) Given $f(x)=\begin{cases} x^2-2x-1 & \text{if} \quad x<3 \\ x+2 & \text{if} \quad x\geq 3 \end{cases}$

(i) Determine $\lim\limits_{x\to 3^-} f(x)$ and $\lim\limits_{x\to 3^+} f(x)$.

[***For this part, you must show at least one intermediate step where*** $\lim\limits_{x\to 3^-}$ ***or*** $\lim\limits_{x\to 3^+}$ ***is still needed.***]

(ii) Does $\lim\limits_{x\to 3} f(x)$ exist? Give your reason. If it exists, state its value.

(iii) Is the function $f(x)$ continuous at $x=3$? Give your reason for your answer.

[4 marks]

(c) (i) State the **intermediate value theorem** (i.e., the full statement including the hypothesis and the conclusion).

(ii) Show that there is a root of the equation $x^4-2x^2+x-3=0$ between 1 and 3. You must write proper steps to arrive at conclusion; just writing some calculations would not be enough.

[3.5 marks]

Continued…….

Question 2 (10 marks)

(a) Use the formal definition of the derivative to compute $f'(2)$ when $f(x) = x^2(x+2)$. ***You are reminded to write proper steps.***

[2.5 marks]

(b) Find $\dfrac{dy}{dx}$ with y as given.

[***Use the product rule or the quotient rule for differentiation; show proper steps.***]

(i) $y = \sqrt{x}\sin 3x$

(ii) $y = \dfrac{e^{2x}}{\tan 3x}$

[3 marks]

(c) The point (2, 1) lies on the curve $2y^2 + 3xy - x^3 = 0$.

Use **implicit differentiation** to obtain $\dfrac{dy}{dx}$ **in terms of** x **and** y.

Then determine the gradient of the tangent to the curve $2y^2 + 3xy - x^3 = 0$ at the point (2, 1).

[4.5 marks]

Continued…….

Question 3 (10 marks)

(a) Solve the following integral using **integration by substitution** $u = \cos x$.

$$\int \frac{\sin x \cos x}{1+\cos x}\,dx$$

[2.5 marks]

(b) (i) Use $\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$ to find the values of A and B which make the equation

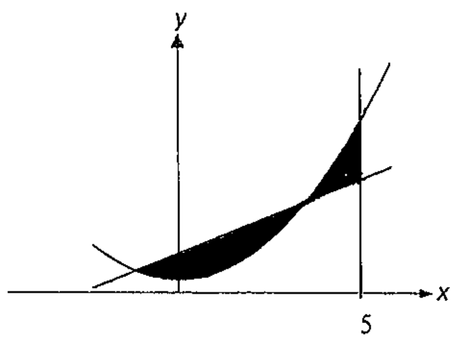
$\cos^3\theta = A\cos 3\theta + B\cos\theta$ an identity.

[*The identity* $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ ***may be useful***]

(ii) Then evaluate $\int_0^{\frac{\pi}{6}} \cos^3\theta d\theta$.

[4 marks]

(c) The following figure shows two regions bounded by parabola $y = 2x^2 + 3$, straight line $y = 4x + 9$ and vertical line $x = 5$.



(i) Determine the x-coordinates of the points of intersection between the parabola and the straight line.

(ii) Find the total area of the bounded region.

[3.5 marks]

Continued.......

Question 4 (10 marks)

(a) Given a sequence $\{a_n\}$ with $a_n = \dfrac{2n^3 - 5n}{3n^3 + 1}$.

(i) Determine $\lim_{n\to\infty} a_n$. You are reminded to write proper steps.

(ii) Then determine whether the infinite series $\sum_{n=1}^{\infty} \dfrac{2n^3 - 5n}{3n^3 + 1}$ is convergent. Give the reason for your answer.

[2 marks]

(b) Determine whether the geometric series $\sum_{n=1}^{\infty} \dfrac{1}{e^n}$ is convergent. If the series is convergent, find its sum.

[1.5 marks]

(c) Find the Maclaurin polynomial of order 3 for $f(x) = x^3 - 2e^{-x}$.

[3 marks]

(d) A periodic function f with period 2π is defined as

$$f(x) = \begin{cases} 0 & -\pi \le x < 0 \\ x & 0 \le x < \pi \end{cases}$$

The **Fourier series** of f has the form $f(x) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$.

Find a_2.

[3.5 marks]

Continued……

Question 5 (10 marks)

(a) Given $F(x, y) = y^3 \sin(x) + \cos(y) + 3x$, find the **partial derivatives** $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$.

[1 mark]

(b) Solve the first order **separable equation** $\frac{dy}{dx} = \frac{x^2 + 1}{2y}$ subject to the initial condition $y(3) = 3$. You may leave your answer in implicit form.

[2 marks]

(c) You are told that e^{-5x} is an integrating factor for the first order linear equation $\frac{dy}{dx} - 5y = e^x$ subject to the initial condition $y(0) = 1$.

Solve the equation and give your solution in explicit form.

[3 marks]

(d) Consider the second order differential equation

$$y'' - 3y' + 2y = 4 + x$$

(i) Find the roots of the **characteristic equation** of the corresponding homogeneous differential equation. Then write down the general solution y_h of this homogeneous differential equation.

(ii) If $y = Ax + B$ is a **particular solution** of the differential equation $y'' - 3y' + 2y = 4 + x$. Determine the values of A and B.

(iii) Hence, write down the **general solution** for the differential equation $y'' - 3y' + 2y = 4 + x$.

[4 marks]

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